THE HEATING WITH GAS OF A FIXED TWO-COMPONENT LAYER WITH VARIOUS INITIAL TEMPERATURES

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We have derived the equations for the change in temperatures in a nonmoving layer consisting of two materials with various thermophysical properties and initial temperatures. An approximate method is proposed for the solution of the problem, based on the utilization of the Schumann graph data.

It is standard practice in blast-furnace operations to charge the furnace with cold coke and hot agglomerate. The temperature of the latter occasionally exceeds 500° C. The heat treatment of the first batch may be regarded in the combined charging of the agglomerate and coke as the gas heating of a two-component layer with various initial temperatures. The quantitative relationships of heat transfer for this and similar cases can be found after the solution of the following problem.

A fixed bed [layer] of height H, consisting of two components with nonidentical thermophysical properties which remain constant during the heating process, is flushed by a gas having a constant temperature at the inlet to the layer. One of the materials prior to the flushing exhibits a temperature different from zero. In view of this circumstance, and also because of the fact that the rate of heating for each of the materials is defined by the magnitude of the surface referred to a unit of material heat capacity at constant volume, a mutual exchange of heat arises between the two materials, described in the general case by the laws of radiation. The problem considered below is linearized by replacing with its constant value the coefficient of mutual heat transfer which changes during the heating process [1].

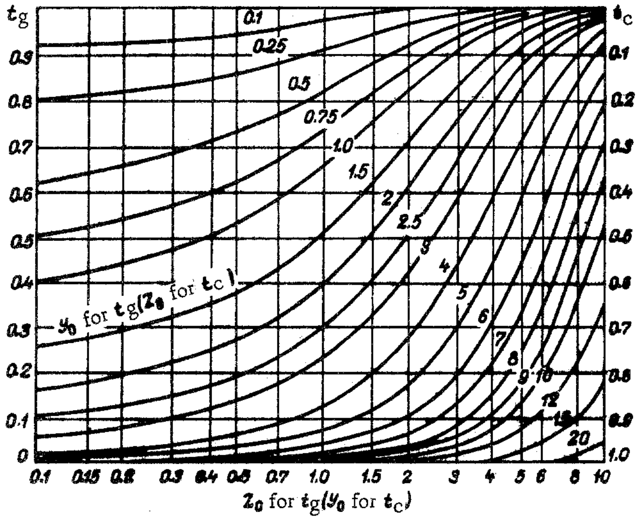

Fig. 1. Plot for determining temperatures of gas and charge in fixed bed.

With this linearization of the boundary condition in the formulated problem, it is necessary to find the time variations in the temperatures of each of the materials and of the gas at any level within the layer.

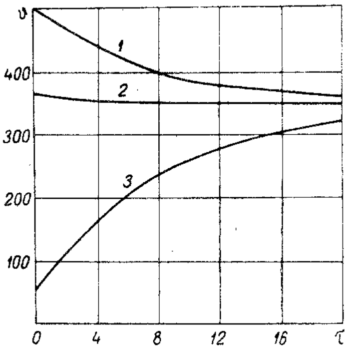

Fig. 2. Temperature history of agglomerate (1), gas (2), and coke (3).

The magnitude of the internal thermal resistance of the particles making up the layer can be neglected, since in the majority of cases the value of the Biot number for these particles does not exceed 0.5.

Examining an elementary layer of height dH, we derive the following equations describing the transfer of heat between the gas and the materials:

for the first material

$$\frac{\partial t_1}{\partial Z} = (t_g - t_1) + A(t_2 - t_1), \tag{1}$$

for the second material

$$\frac{\partial t_2}{mn\partial Z} = (t_g - t_2) - A(t_2 - t_1)\frac{1}{n}, \tag{2}$$

for the gas flow

$$-\frac{\partial t_g}{\partial Y} = (t_g - t_1) + (t_r - t_2)\,n, \tag{3}$$

and the boundary conditions of the problem

$$Y = 0, \quad t_g = T, \tag{4}$$

$$Z = 0, \quad t_1 = 0, \quad t_2 = t_0. \tag{5}$$

The over-all solution for the problem will be sought in the form of sums of solutions, and namely:

$$t_1 = t_1' + t_1'', \qquad t_2 = t_2' + t_2'', \qquad t_g = t_g' + t_g'', \tag{6}$$

t_i' in these satisfies the boundary conditions

$$Y = 0, \; t_g' = T \text{ and } Z = 0, \; t_1' = t_2' = 0\,; \tag{7}$$

and t''_i

$$Y=0,\ t''_g=0 \text{ and } Z=0,\ t''_1=0,\ t''_2=t_0. \tag{8}$$

It is natural that t'_i and t''_i satisfy Eqs. (1)–(3).

The solution of the problem satisfying boundary conditions (7) was derived by us in [1]; it is the purpose of the present paper to seek a solution which would satisfy conditions (8).

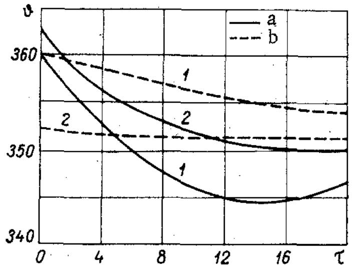

Fig. 3. Temperature history obtained as a result of calculations according to the proposed methods (a) and the usually applied methods (b): 1) charge (average weight); 2) gas.

Applying the Laplace transform in Z to Eqs. (1) and (2), with consideration of (8), we obtain

$$p\bar{t}''_1=\bar{t}''_g-\bar{t}''_1+A(\bar{t}''_2-\bar{t}''_1), \tag{1'}$$

$$p\bar{t}''_2-t_0=mn(\bar{t}''_g-\bar{t}''_2)-Am(\bar{t}''_1-\bar{t}''_2), \tag{2'}$$

whence $\bar{t}''_1$ and $\bar{t}''_2$ is found as a function of $\bar{t}''_g$:

$$\bar{t}''_1=\frac{\bar{t}''_g(p+mn+Amn+Am)+At_0}{(p+1+A)(p+mn+Am)-A^2m}, \tag{9}$$

$$\bar{t}''_2=\frac{\bar{t}''_g[mn(p+1+A)+Am]+(p+1+A)t_0}{(p+1+A)(p+mn+Am)-A^2m}. \tag{10}$$

The representation of the gas-flow equation after the transformations, with consideration of expressions (9) and (10), assumes the form

$$-\frac{d\bar{t}''_g}{dY}=\left(1+n-\frac{a}{p-R_1}-\frac{b}{p-R_2}\right)\bar{t}''_g-$$
$$-\frac{A(1+n)+n(1+p)}{(p-R_1)(p-R_2)}\,t_0, \tag{11}$$

where

$$a=\frac{(k+R_1)(1+n^2m)}{R_1-R_2};\quad b=\frac{(k+R_2)(1+n^2m)}{R_2-R_1};$$

$$k=\frac{mn(1+n)+Am(1+n)^2}{1+n^2m};$$

$$R_{1,2}=-\frac{1}{2}(1+A+Am+mn)\pm$$
$$\pm\frac{1}{2}\sqrt{(1+A+Am+mn)^2-4[mn+Am(1+n)]}.$$

The solution of (11) after satisfaction of boundary conditions (8) is as follows:

$$\bar{t}''_r=t_0\frac{A(1+n)+n(p+1)}{p[(1+n)p+(m+1)(n+A+An)]}\times$$
$$\times\left\{1-\exp\left[-\left(1+n-\frac{a}{p-R_1}-\frac{b}{p-R_2}\right)Y\right]\right\}. \tag{12}$$

The original of the fractional factor in (12), or what is the same, of the first term, can be found by using the theorem of expansion. With this aim in mind, we have to determine the roots of the denominator of the fraction, and these, as is not difficult to see, are equal to $p_1=0$ and $p_2=-\frac{m+1}{n+1}(n+A+An)$. Further, applying the theorem of multiplication, we obtain the expression for the original of the function t''_g:

$$t''_g=t_0\left[\frac{1}{1+m}-\frac{1-mn}{(1+m)(1+n)}e^{BZ}\right]-$$
$$-t_0\int_0^Z t'_g(Y,\varepsilon)\frac{1-mn}{(1+n)^2}\times$$
$$\times(n+A+An)e^{B(Z-\varepsilon)}d\varepsilon-$$
$$-t_0\frac{n}{1+n}t'_g(Y,Z), \tag{13}$$

where $B=-\frac{1+m}{1+n}(n+A+An)$, and $t'_g(Y,Z)$ is the known solution of (1)–(3) for boundary conditions (7) and T = 1 [1].

In this connection it becomes possible to write the over-all solution in the form of the sum of two solutions (see (6)). In final form the equation describing the changes in gas temperature with time and in layer height will be:

$$t_g=t_0\left[\frac{1}{1+m}-\frac{1-mn}{(1+m)(1+n)}e^{BZ}\right]-$$
$$-Ce^{-(1+n)Y}\int_0^Z e^{B(Z-\varepsilon)}\left\{e^{R_1\varepsilon}I_0(2\sqrt{aY\varepsilon})-\right.$$
$$-\int_0^\varepsilon R_1e^{R_1\nu}I_0(2\sqrt{aY\nu})\,d\nu+\int_0^\varepsilon[e^{R_1x}I_0(2\sqrt{aYx})-$$
$$\left.-\int_0^x R_1e^{R_1\nu}I_0(2\sqrt{aY\nu})\,d\nu\right]\times$$
$$\left.\times\sqrt{\frac{bY}{\varepsilon-x}}e^{R_2(\varepsilon-x)}I_1(2\sqrt{bY(\varepsilon-x)})\,dx\right\}d\varepsilon+$$
$$+\left(T-\frac{n}{1+n}t_0\right)e^{-(1+n)Y}\left\{e^{R_1Z}I_0(2\sqrt{aYZ})-\right.$$
$$-\int_0^Z R_1e^{R_1\varepsilon}I_0(2\sqrt{aY\varepsilon})\,d\varepsilon+$$
$$+\int_0^Z\left[e^{R_1x}I_0(2\sqrt{aYx})-\right.$$
$$\left.-\int_0^x R_1e^{R_1\varepsilon}I_0(2\sqrt{aY\varepsilon})\,d\varepsilon\right]\times$$
$$\left.\times\sqrt{\frac{bY}{Z-x}}e^{R_2(Z-x)}I_1(2\sqrt{bY(Z-x)})\,dx\right\}, \tag{14}$$

where

$$C = \frac{1-mn}{(1+n)^2}\,(n+A+An).$$

The temperature of the first material can be found from (1), (2), and (3) on the assumption that t_g is a known quantity. The expression for the calculation of t_1 then assumes the form

$$t_1 = \int_0^Z \left(Et_g + \frac{A}{n}\,\frac{\partial t_g}{\partial Y}\right)\exp\left[E(\varepsilon - Z)\right]. \qquad (15)$$

For t_2, we can obtain analogously

$$t_2 = t_0 \exp\left[-mnEZ\right] +$$
$$+ mn \int_0^Z \left(Et_g + \frac{A}{n}\,\frac{\partial t_g}{\partial Y}\right)\exp\left[mnE(\varepsilon - Z)\right]d\varepsilon, \qquad (16)$$

where

$$E = 1 + A\,\frac{1+n}{n}.$$

In calculating t_1 and t_2 we must know t_g and $\partial t_g/\partial Y$. The first quantity is determined from (14), while the second can be found by differentiation of (14) with respect to Y.

Expressions (14)–(16) represent the over-all solution to the problem, satisfying (1)–(3) with boundary conditions (4) and (5). Calculations of the temperature field in the nonmoving layer are facilitated by the knowledge that the terms of the expressions for the determination of $\partial t_g/\partial Y$ and (14) in a number of cases consist of identical functions.

It is interesting to note that unlike the single-component layer and the two-component layer with identical initial temperatures, the temperature of the gas at points distant from the point of gas entry into the layer is not a constant but a function of time. This function is expressed by the formula

$$t_g = t_0\left[\frac{1}{1+m} - \frac{1-mn}{(1+m)(1+n)}\exp(BZ)\right] \qquad (17)$$

and is explained by the transfer of heat from the hot material to the cold at any point in the layer. At the initial instant of time at these points the temperature of the gas is equal to the average weighted value over the surface:

$$t_g = \frac{n}{1+n}\,t_0. \qquad (18)$$

The approximate calculations of temperature distributions in the layer can be carried out on the basis of the data from the Schumann graphs [2, 3]. For this t_g and $\partial t_g/\partial Y$ should be written in the form

$$t_g = t_0\left[\frac{1}{1+m} - \frac{1-mn}{(1+m)(1+n)}\,e^{BZ} -\right.$$
$$\left. - \frac{1+m}{m(1+n)}\,C\int_0^{Z_0} t_g^0(Y_0, \varepsilon)\,e^{B\left[Z - \frac{1+m}{m(1+n)}\varepsilon\right]}d\varepsilon\right] +$$
$$+ \left(T - \frac{n}{1+n}\,t_0\right) t_g^0(Y_0, Z_0) \qquad (19)$$

and

$$\frac{\partial t_g}{\partial Y} = (1+n)\left[\left(T - \frac{n}{1+n}\,t_0\right)\frac{\partial t_g^0}{\partial Y_0} -\right.$$
$$\left. - \frac{1+m}{m(1+n)}\,Ct_0\int_0^{Z_0}\frac{\partial t_g^0}{\partial Y_0}\,e^{\left[Z - \frac{1+m}{m(1+n)}\varepsilon\right]}d\varepsilon\right], \qquad (20)$$

where t_g^0 is the temperature of the gas in the layer, determined from the Schumann graphs for $Y_0 = (1+n)Y$ and $Z_0 = \frac{m(1+n)}{1+m}Z$ for $T = 1$. With this approach to the solution of the problems for a nonmoving layer, with loss of some accuracy, it becomes possible to reduce considerably the volume of calculations. The magnitude of the error in this case, as demonstrated by numerical analysis, for all $Y_0 < 6.0$, does not exceed 0.6% in the determination of the gas and material temperatures.

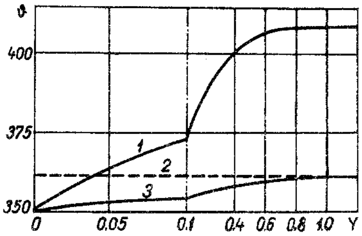

Fig. 4. Temperature distribution with respect to gas layer height at initial time instant calculated according to the proposed methods (1), usually applied methods (3) as well as according to agglomrate and coke (average weight (2).

For the calculations according to the proposed method it is necessary to have the Schumann graphs available in addition to the data on the changes in the material and gas temperatures in the region of low values of Y_0 and Z_0, and in such quantities as would permit the relatively exact determination of the quantity $\partial t_g^0/\partial Y_0$. In this connection, the authors calculated the curves $t_g^0 = f(Y_0, Z_0)$ for Y_0, equal to 0.1, 0.25, 0.75, 1.5, and 2.5. These curves have been plotted on the Budrin graph [3] and this graph is convenient in that, first of all, it covers both the gas and the material and, secondly, it is constructed in semilogarithmic coordinates as a result of which it encompasses a wide range of values for Y_0 and Z_0. This combined graph is shown in Fig. 1.

With regard to the subject problem the coefficient α_{rad} of mutual heat transfer must be calculated according to the formula

$$\alpha_{rad} = 4\,\frac{C_p}{100}\left(\frac{\overline{T}}{100}\right)^3 \varphi_{21} f_2, \qquad (21)$$

in which $\overline{T}$ in °K represents the mean temperature of the system before the flushing, i.e.,

$$\bar{T}=\frac{1}{2}\left(\vartheta_{0g}+\frac{\vartheta_{01}\varphi_1 c_1\gamma_1+\vartheta_{02}\varphi_2 c_2\gamma_2}{\varphi_1 c_1\gamma_1+\varphi_2 c_2\gamma_2}\right)+273. \quad (22)$$

The resulting solution was used to calculate the gas and material temperatures in the charge layer loaded into the blast furnace. The charge consists of coke and the agglomerate. The characteristics of the charge as applicable to contemporary blast furnaces are the following: the subscript 1 pertains to the coke and the subscript 2 pertains to the agglomerate; $\varphi_1 = \varphi_2 = 0.5$; $f_1 = 30.6$ m^2/m^3; $f_2 = 120$ m^2/m^3; n = 3.92; $c_1 = 1.203$ kJ/kg · deg; $c_2 = 0.891$ kJ/kg· deg; $c\gamma = 1.468$ kJ/m^3·deg; $\gamma_1 = 600$ kg/m^3; $\gamma_2 = 1800$ kg/m^3; m = 0.45; w = 5720 m/hr; $\alpha = 1152$ W/m^3 · deg. We are considering the case of transition to furnace operation with hot agglomerate, when its initial temperature is equal to 500° C and the coke temperature is 50° C. The height of the loaded charge portion generally is 0.5 m. The gas temperature at the inlet to the layer is 350° C. The coefficient of mutual heat transfer between the agglomerate and the coke by definition is equal to 331 W/m^3· deg.

The results from the calculation of the time variations of the temperatures for the two materials and for the gas at the outlet from the layer are shown in Fig. 2. The data of the drawing indicate the relatively rapid heating of the coke and the cooling of the agglomerate. This shape for the curves is governed by the intensive development of the heat-transfer processes, primarily between the agglomerate and the coke. The agglomerate under the given conditions plays the role of the principal heat carrier. Within the first 16 min of the process, approximately twice the heat is received from the agglomerates and from the gas. Moreover, the agglomerate transfers a substantial quantity of heat to the gas, a quantity which is larger than the quantity of heat given off by the gas to the coke. As a result of this relationship of heat flows, an excess of heat arises in the gas, and this is carried out of the layer. Nevertheless, it should be noted that the main portion of the heat of the agglomerate (95% of its heat content on loading) remains in the layer, which has an extremely favorable effect on the development of the technological process.

There is some interest in comparing the result from the determination of temperatures according to the proposed method with the method normally employed, when the calculations are carried out for a layer of equivalent heat capacity with an initial temperature equal to the weighted mean. This comparison for the outlet cross section of the layer is shown in Fig. 3, and from this it follows that the calculations of a single example by either of the methods lead to fundamentally divergent curves for the time-variations in temperature. The variations in the temperature curves found by the second method indicate the continuous cooling of the layer by the gas, with the temperature of the charge at no time dropping below the gas temperature at the inlet to the layer.

At the same time, the calculations according to the proposed method reveal a more complex pattern. First of all, at $\tau = 0$ the gas is heated rapidly because of the hot agglomerate and even at a distance of 0.4 m from the inlet to the layer reaches a weighted mean for the charge temperature, calculated from the temperatures of the individual components. If the layer exhibited a height of 5m (Y = 0.5), the gas temperature at the outlet of the layer at this instant would exceed the initial temperature by 53.4° C (Fig. 4). Such intense thermal action by the agglomerate on the gas is governed by the relatively high specific surface of the agglomerate per 1 m^3 of layer and its initial temperature. Secondly, the weighted mean temperature of the charge during the first half of the process diminishes, and 5 min after the beginning of the flushing it drops below the temperature of the gas at the inlet to the layer. Only after the temperature reaches the initial value as a result of the gas-flow cooling, does the weighted mean temperature of the charge begin to rise as a result of the continuing heating of the colder charge component, i.e., of the coke. Consequently, the leveling of the temperatures in the layer for the subject conditions of the example takes place only after the "supercooling" of the charge.

Thus, calculations according to the proposed method make it possible to establish a number of features in the dynamics of the process involved in the heating of a two-component charge with various initial temperatures, and these cannot be revealed through the conventional calculations carried out for a charge of equivalent heat capacity.

NOTATION

t_i and t_g are the excess temperatures; ϑ_i, ϑ_g, and ϑ_{01} are the instantaneous and initial temperatures of the first material; Y is the criterion of layer height; Z is the time criterion; H is the layer height; w is the gas velocity; τ is the time; φ_i is the material fraction in 1 m^3 of a layer; f_i is the material surface in 1 m^3 of a layer; $n = f_2/f_1$ is the ratio of material surfaces; c_i and c are the mass heat capacity of material and gas; γ_i and γ are the mass of bulk of material and gas density; m is the ratio of volume heat capacities for materials; α is the coefficient of heat transfer from gas to material surface; α_{rad} is the coefficient of mutual heat transfer between materials with different thermal properties and initial temperatures; C_p is the radiation coefficient; φ_{21} is the angle factor from second material to first; T is the mean temperature of a system before injection; A is the ratio of heat transfer coefficients; subscripts 1 and 2 refer to the first and second materials, respectively.

SUMMARY

In the paper the system of equations (1)–(5) is solved which describes gas heating of a fixed layer consisting of two materials with different thermal properties and initial temperatures. The general solution to the problem is found as a sum (6) every summand of which satisfies boundary conditions (7) and (8), respectively. The equations obtained in the final form allow prediction of the change in the tem-

peratures of the gas and every material (15), (16) depending on the time and the height of the layer. An approximate method is proposed for calculation of the temperature distribution in the layer based on application of the data from the Shumann graphs which are supplemented with curves in the region of low numbers of the layer height. The problem solution is illustrated by an example (Fig. 2).

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